MATH 135 — QUIZ 4 — JAMES HOLLAND

2019-10-01

Question 1. Consider the function $f(x) = 3\cos^2(x)$.

i. Calculate $f'(x)$ using the product rule.

ii. Calculate $f'(x)$ using the chain rule.

Solution .:.

i. $f(x)$ can be written as the product $3\cos(x) \cdot \cos(x)$. Take $g(x) = 3\cos(x)$ and $h(x) = \cos(x)$ so that

$$g(x) = 3\cos x \qquad h(x) = \cos x$$
$$g'(x) = -3\sin x \qquad h'(x) = -\sin x.$$

The product rule states the equality

$$\begin{aligned} f'(x) = (g \cdot h)'(x) &= g'(x)h(x) + g(x)h'(x) \\ &= -3\sin(x) \cdot \cos x + 3\cos(x)(-\sin x) \\ &= -6\sin x \cos x. \end{aligned}$$

ii. $f(x)$ can be decomposed as the composition between $g(x) = 3x^2$ and $h(x) = \cos x$: $f(x) = g(h(x))$. Note that then

$$g(x) = 3x^2 \qquad h(x) = \cos x$$
$$g'(x) = 6x \qquad h'(x) = -\sin x.$$

By the chain rule, we get the same answer as in (i):

$$f'(x) = g'(h(x)) \cdot h'(x) = 6\cos x \cdot (-\sin x) = -6\sin x \cos x.$$

Question 2. Consider $f(x) = \dfrac{e^x}{2x^4 - x}$, and $g(x) = \ln(5x)$.

i. Calculate $f'(x)$.

ii. Calculate $g'(x)$.

Solution .:.

i. We can write $f(x) = h(x)/k(x)$ for $h(x) = e^x$ and $k(x) = 2x^4 - x$ so that

$$h(x) = e^x \qquad k(x) = 2x^4 - x$$
$$h'(x) = e^x \qquad k'(x) = 8x^3 - 1.$$

Using the quotient rule (or equivalently the chain and product rules with $h(x) \cdot \frac{1}{k(x)}$), we have

$$f'(x) = \frac{h'(x) \cdot k(x) - h(x) \cdot k'(x)}{k(x)^2} = \frac{e^x \cdot (2x^4 - x) - e^x(8x^3 - 1)}{(2x^4 - x)^2} = \frac{e^x \cdot (2x^4 - 8x^3 - x + 1)}{(2x^4 - x)^2}.$$

ii. There are two easy ways of evaluating this derivative. Firstly, $g(x) = \ln(5) + \ln(x)$. Since $\ln(5)$ is a constant, it's derivative is 0, and so $g'(x) = 0 + \frac{\mathrm{d}}{\mathrm{d}x}\ln(x) = \frac{1}{x}$.

Another way to evaluate the derivative is through the chain rule. In particular,

$$g'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\ln(5x) = \frac{1}{5x} \cdot \frac{\mathrm{d}}{\mathrm{d}x} 5x = \frac{1}{5x} \cdot 5 = \frac{1}{x}.$$